$x \text{ XOR } y \text{ XOR } z$

| | |
|---|---|
| $= (x \text{ XOR } y) \text{ XOR } z$ | by associativity |
| $= (x \text{ XOR } y)'z + (x \text{ XOR } y)z'$ | from Figure 1-2 |
| $= (x'y+xy')'z + (x'y+xy')z'$ | from Figure 1-2 |
| $= ((x'y)' \cdot (xy')')z + x'yz' + xy'z'$ | from rule 15 and 13 |
| $= ((x+y') \cdot (x'+y))z + x'yz' + xy'z'$ | from rule 16 |
| $= (xx'+xy+x'y'+yy')z+x'yz + xy'z'$ | by FOIL |
| $= xx'z + xyz + x'y'z +yy'z + x'yz' + xy'z'$ | from rule 13 |
| $= \quad 0 \quad + xyz + x'y'z + 0 \quad +x'yz' + xy'z'$ | from rule 8 |
| $= \quad xyz + x'y'z + x'yz' + xy'z'$ | by rule 1 |

Note that each term has an odd number of unprimed elements.

Take the K-map for S

| | | | |
|---|---|---|---|
| | 1 | | 1 |
| 1 | | 1 | |

The K-map is exactly $xyz + x'y'z + x'yz' + xy'z'$ which we just proved equals $x \text{ XOR } y \text{ XOR } z$.